

# Package: equate (via r-universe)

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**Type** Package

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**Title** Observed-Score Linking and Equating

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**URL** <https://github.com/talbano/equate>

**BugReports** <https://github.com/talbano/equate/issues>

**Depends** R (>= 4.1.0)

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**Description** Contains methods for observed-score linking and equating under the single-group, equivalent-groups, and nonequivalent-groups with anchor test(s) designs. Equating types include identity, mean, linear, general linear, equipercentile, circle-arc, and composites of these. Equating methods include synthetic, nominal weights, Tucker, Levine observed score, Levine true score, Braun/Holland, frequency estimation, and chained equating. Plotting and summary methods, and methods for multivariate presmoothing and bootstrap error estimation are also provided.

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**LazyData** yes

**License** GPL-3

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**VignetteBuilder** knitr

**Repository** <https://talbano.r-universe.dev>

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equate-package	<i>Observed-Score Linking and Equating</i>
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## Description

Contains methods for observed-score linking and equating under the single-group, equivalent-groups, and nonequivalent-groups with anchor test(s) designs. Equating types include identity, mean, linear, general linear, equipercntile, circle-arc, and composites of these. Equating methods include synthetic, nominal weights, Tucker, Levine observed score, Levine true score, Braun/Holland, frequency estimation, and chained equating. Plotting and summary methods, and methods for multivariate presmoothing and bootstrap error estimation are also provided.

## Details

Package: equate  
 Version: 2.0.8  
 Date: 2022-06-05  
 Depends: R (>= 4.1.0)  
 License: GPL-3

### Index:

ACTmath	ACT Mathematics Test Scores
KBneat	Test Scores Under a NEAT design
PISA	Programme for International Student Assessment 2009 USA Data
descriptives	Descriptive Statistics for Frequency Tables

percentiles	Percentile Ranks and Cumulative Frequencies for Frequency Tables
equate	Observed Score Linking and Equating
composite	Composite Linking and Equating
bootstrap	Bootstrap Equating Error
freqtab	Frequency Distribution Tables
presmoothing	Frequency Distribution Presmoothing
plot.freqtab	Plotting Frequency Distributions
plot.equate	Plotting Equating Results

Further information is available in the following vignettes:

`equatevignette`    `equate vignette (source, pdf)`

The package vignette provides an introduction to linking and equating and includes descriptions of the supported equating methods and examples. The help page for the main function of the package, `equate`, contains additional examples.

#### Author(s)

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ACTmath

*ACT Mathematics Test Scores*

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#### Description

This dataset contains score distributions for two forms of the ACT mathematics test, as presented in table 2.5 of *Test Equating, Scaling, and Linking* (Kolen and Brennan, 2004; p. 50).

#### Usage

ACTmath

#### Format

A 41 by 3 data.frame containing the score scale, frequencies for form X, and frequencies for form Y.

#### Source

Kolen, M. J., and Brennan, R. L. (2004). *Test Equating, Scaling, and Linking*. (2nd ed.), New York: Springer.

The dataset is also provided with the equating software RAGE, available at the following link: <https://education.uiowa.edu/casma/computer-programs>

bootstrap

*Bootstrap Equating Error***Description**

These functions return bootstrap standard errors, bias, and RMSE of equating. A summary method estimates mean and weighted mean errors over the score scale.

**Usage**

```
bootstrap(x, ...)

## Default S3 method:
bootstrap(x, y, ...)

## S3 method for class 'equate'
bootstrap(x, xp = x$x, yp = x$y, ...)

## S3 method for class 'freqtab'
bootstrap(
  x,
  y,
  xn = sum(x),
  yn = sum(y),
  reps = 100,
  crit,
  args,
  eqs = FALSE,
  sharesmooth = FALSE,
  ...
)

## S3 method for class 'bootstrap'
summary(object, weights, subset, ...)
```

**Arguments**

x	either an equating object, obtained with the <a href="#">equate</a> function, or a score distribution of class “ <a href="#">freqtab</a> ”.
...	further arguments passed to or from other methods.
y	score distribution of class “ <a href="#">freqtab</a> ”.
xp, yp	optional frequency tables replacing those equated in x, used for parametric bootstrap resampling.
xn, yn	integers specifying the number of scores to sample from each distribution at each replication (default is the total number observed in each).

reps	number of bootstrap replications.
crit	vector of equated scores serving as the criterion equating function when calculating bootstrap bias and RMSE, both of which are returned when crit is specified.
args	named list of equating arguments, passed to <code>equate</code> , specifying, e.g., the equating type and method. See below for details.
eqs	logical, with default FALSE, indicating whether or not the matrices of equating functions (one column per replication, per equating) should be returned.
sharesmooth	logical, defaulting to FALSE, indicating whether or not loglinear presmoothing should be performed once per replication using arguments given in args. Ignored if <code>smoothmethod = "loglinear"</code> is not given in any args.
object	bootstrap output to be summarized.
weights	vector of weights to be used in calculating weighted average errors with summary, defaulting to the frequencies in <code>margin(object\$x)</code> .
subset	vector indicating a subset of the score scale for which errors should be summarized.

### Details

Samples are drawn of size  $x_n$  and  $y_n$ , with replacement, from each score distribution. Form  $Y$  equivalents of each form  $X$  score are then obtained using either the arguments in the equating output or those provided. This process is repeated `reps` times. Standard errors are calculated as standard deviations over replications for each score point; bias is the mean equated score over replications, minus the criterion; and RMSE is the square root of the squared standard error and squared bias combined.

The bootstrap method for objects of class “`equate`” is designed to be called from within `equate`. It simply extracts the necessary arguments from the equating output before bootstrapping.

When each element in `args` is a named list of equating arguments, multiple equatings are performed at each replication in the bootstrapping.

The summary method returns a `data.frame` of mean standard errors, bias, and `rmse`, and weighted means, as applicable.

### Value

With `bootstrap`, a list is returned, containing arguments supplied for `x`, `y`, `reps`, `xn`, `yn`, and `args`. For a single equating, the mean equating function over replications and a vector of standard errors `se` are included, along with vectors of `bias` and `rmse`, when `crit` is provided, and a matrix of equating functions `eqs` when `eqs = TRUE`. For multiple equatings, where each element of `args` is a list of equating arguments, matrices are returned for the mean functions, standard error, bias, and RMSE, and the equating functions will be returned as a list of matrices. The summary method returns a `data.frame` of mean standard errors, bias, and `rmse`, and weighted means, as applicable.

### Methods (by class)

- `bootstrap(default)`: Default bootstrap method for “`freqtab`” objects.
- `bootstrap(equate)`: Method for “`equate`” objects.
- `bootstrap(freqtab)`: Bootstrap method for “`freqtab`” objects.

**Author(s)**

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**See Also**

[plot.bootstrap](#)

**Examples**

```
# Parametric bootstrapping using smoothed
# frequency distributions
set.seed(111213)
x <- freqtab(KBneat$x, scales = list(0:36, 0:12))
y <- freqtab(KBneat$y, scales = list(0:36, 0:12))
xp <- loglinear(x, asfreqtab = TRUE)
yp <- loglinear(y, asfreqtab = TRUE)
crit <- equate(xp, yp, "e", "c")$conc$yx
eqargs <- list(m.t = list(type = "m", method = "t"),
  l.t = list(type = "l", method = "t"))
bootout1 <- bootstrap(x = x, y = y, xn = 20, yn = 20,
  crit = crit, args = eqargs, reps = 30)
plot(bootout1, out = "rmse", legendplace = "top",
  addident = FALSE)

# Bootstraps for an existing equating
eq <- equate(x, y, type = "m", method = "t")
bootout2 <- bootstrap(eq, xn = 100, yn = 100,
  crit = crit, reps = 20)
summary(bootout2)
```

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composite

*Composite Linking and Equating*

---

**Description**

This function creates a composite linking or equating as a combination of two or more other linking or equating functions.

**Usage**

```
composite(x, ...)

## Default S3 method:
composite(x, wc, ...)

## S3 method for class 'equate.list'
composite(x, wc, name, symmetric = FALSE, p = 1, verbose = TRUE, ...)
```

```
## S3 method for class 'list'
composite(x, wc, name, symmetric = FALSE, p = 1, verbose = TRUE, ...)
```

### Arguments

x	for the default method, x is a matrix of equating functions, with one function per column. Otherwise, x is a list of equatings, where each element is an object of class “ <a href="#">equate</a> ”.
...	further arguments passed to or from other functions.
wc	vector of weights for creating the composite. <code>length(wc)</code> should match either <code>ncol(x)</code> for the default method or <code>length(x)</code> .
name	an optional name, used to label the output. If missing, a name will be created using x.
symmetric	logical, with default FALSE, indicating whether or not weights wc should be modified to create symmetric weights. Only supported for composites of linear functions.
p	integer specifying the type of circle used to define symmetry.
verbose	logical, with default TRUE, indicating whether or not full output should be returned. When FALSE, only the equated scores are returned.

### Details

Composite linking and equating create a single linking or equating function as a weighted combination of two or more other linking or equating functions. See Holland and Strawderman (2011) for details.

### Value

For the default method, and when `verbose = FALSE`, a vector of composite equated scores is returned. Otherwise, a list of equating output is returned, including output for the composite and each function being combined.

### Methods (by class)

- `composite(default)`: Default method for a matrix of equating functions, one per column.
- `composite(equate.list)`: Create composite across functions in “`equate.list`” object.
- `composite(list)`: Create composite across functions in “`list`” object.

### Author(s)

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### References

Holland, P. W., and Strawderman, W. E. (2011). How to average equating functions, if you must. In A. A. von Davier (Ed.), *Statistical models for test equating, scaling, and linking* (pp. 89-107). New York, NY: Springer.

**See Also**[equate](#)**Examples**

```
# See vignette("equatevignette") for additional examples

# Example from the equate help file, without the bootstrapping
# Random groups equating for (1) identity, (2) mean,
# (3) linear, (4) equipercentile with loglinear
# smoothing, and (5) a composite of mean and identity
rx <- as.freqtab(ACTmath[, 1:2])
ry <- as.freqtab(ACTmath[, c(1, 3)])
set.seed(2007)

req1 <- equate(rx, ry, type = "i")
req2 <- equate(rx, ry, type = "m")
req3 <- equate(rx, ry, type = "l")
req4 <- equate(rx, ry, type = "e", smooth = "loglin",
              degrees = 3)
req5 <- composite(list(req1, req2), wc = .5, symmetric = TRUE)

# Compare equating functions
plot(req1, req2, req3, req4, req5[[1]], additnt = FALSE)
```

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equate

*Observed Score Linking and Equating*


---

**Description**

This function links the scale of  $x$  to the scale of  $y$  for the single-group, equivalent-groups, and nonequivalent-groups with anchor test designs. A summary method is also provided.

**Usage**

```
equate(x, ...)

## S3 method for class 'list'
equate(x, args, ...)

## S3 method for class 'freqtab'
equate(
  x,
  y,
  type = c("identity", "mean", "linear", "general linear", "circle-arc",
           "equipercentile"),
  method = c("none", "nominal weights", "tucker", "levine", "frequency estimation",
```



```

    "chained", "braun/holland"),
  name,
  lowp,
  highp,
  boot = FALSE,
  verbose = TRUE,
  ...
)

## Default S3 method:
equate(x, y, ...)

## S3 method for class 'equate'
summary(object, ...)

## S3 method for class 'equate.list'
summary(object, ...)

```

## Arguments

<code>x, y</code>	for the default method, <code>x</code> must be a vector of scores and <code>y</code> an object of class "equate", the output of a previous equating. The standard usage is to provide <code>x</code> as a frequency table of class "freqtab", where <code>y</code> is also a frequency table, and <code>x</code> is equated to <code>y</code> ; if <code>y</code> is missing, a single-group design is assumed. Finally, <code>x</code> may be a list of two or more frequency tables, in which case the required arguments for one or more equatings are listed in <code>args</code> . See below for details.
<code>...</code>	further arguments passed to or from other functions, including arguments specific to different equating methods. See below for details.
<code>args</code>	list of arguments passed to <code>equate</code> . See below for details.
<code>type</code>	the type of equating. See below for details.
<code>method</code>	the equating method, where "none" (default) indicates equating under the single-group or equivalent-groups design, and "nominal weights", "tucker", "levine", "frequency estimation", "braun/holland", and "chained" indicate the corresponding methods under the nonequivalent groups design.
<code>name</code>	an optional name, used to label the output. If missing, a name will be created using <code>type</code> and <code>method</code> .
<code>lowp, highp</code>	two vectors, each of length 2, specifying the coordinates for the low and high points of the X and Y scales. <code>lowp</code> defaults to the minimums and <code>highp</code> the maximums of the scales. Recycled if necessary. When <code>lowp = "obs"</code> or <code>highp = "obs"</code> , minimum and maximum observed scores are used.
<code>boot</code>	logical indicating whether or not bootstrapping should be performed. Default is FALSE. See below and the <code>bootstrap</code> function for details.
<code>verbose</code>	logical, with default TRUE, indicating whether or not full output should be returned. When FALSE, only the equated scores are returned.
<code>object</code>	output from either an equating or list of equatings, produced by the <code>equate</code> function.

## Details

Equating is typically performed on two frequency tables,  $x$  and  $y$ . In this case, the scores from both are used to define the equating function that links the scale of  $x$  to  $y$ . The equivalent-groups design is assumed when  $x$  and  $y$  are separate, univariate frequency tables. The nonequivalent-groups design is assumed when a method is specified, and  $x$  and  $y$  are separate multivariate frequency tables. Finally, a single-group design is assumed when  $x$  is a bivariate frequency table (containing scores on  $X$  and  $Y$ ) and  $y$  is missing.

The single-group design currently only differs from the equivalent groups design in that presmoothing can be used to preserve bivariate moments for  $x$  and  $y$  in the single-group design, whereas in the equivalent-groups design, with  $x$  and  $y$  specified separately, presmoothing is performed separately. If presmoothing is not performed via `equate`, the single-group and equivalent-groups designs produce the same result.

When  $x$  is a vector of scores and equating output is supplied for  $y$ , no other arguments are required. Scores from  $x$  are converted directly to the scale indicated in  $y$ . If  $y$  is a composite equating, composite equated scores will be returned based on the weighted combination of equating functions included in  $y$ .

When  $x$  is a list of frequency tables, each element in `args` must be a named list of equating arguments. In this case, the length of `args` corresponds to the number of equatings that will be performed. The arguments for each equating are specified as they would be when  $x$  and  $y$  are frequency tables, except for  $x$  and  $y$ ; the frequency tables to be equated are specified in `args` by referencing their names in the list of frequency tables. See below for examples.

Six equating types are currently supported: identity, mean, linear, and equipercntile, as described by Kolen and Brennan (2004); circle-arc equating, as described by Livingston and Kim (2009); and a general linear function that extends the traditional identity, mean, and linear types. Corresponding linking methods are also supported. The equating design is implied by the `method` argument, where "none" (default) indicates that no method is needed (because examinees taking forms  $X$  and  $Y$  are assumed to be the same or equivalent). The nominal weights, Tucker, Levine observed score, Levine true score, frequency estimation, Braun/Holland, and chained equating methods are supported for the nonequivalent-groups with anchor test design. All but the Levine true score and chained method rely on a "synthetic" distribution of scores (Braun and Holland, 1982), a weighted combination of  $x$  and  $y$ .

Depending on the equating method, the following additional arguments may be required:

**midp** coordinates for the midpoint of the equating line, used in general linear and circle-arc equating.

**cx, cy, sx, sy** parameters used in general linear equating. See below for details.

**wax, way, wbx, wby** weights used when finding the slope and intercept in general linear equating. See below.

**ws** value between 0 and 1 specifying the weight applied to form  $X$  scores (and implicitly specifying the form  $Y$  weight as  $1 - ws$ ) when estimating the synthetic population. When set to -1 (the default), proportional weights are calculated for  $X$  and  $Y$  based on sample size.

**internal** logical indicating whether or not the anchor item scores are included in the total scores. This applies only to the Levine method, as all other methods assume an internal anchor test. Default is TRUE.

**lts** logical indicating whether or not to use levine true score ("lts") equating. Default is FALSE.

- smoothmethod** string indicating one of four smoothing methods to be used in equipercentile equating: "none" (default), "average", "bump", and "loglinear" (see below).
- chainmidp** string specifying the type of chained linear equating used to obtain the midpoint in chained circle-arc equating, whether "mean" (default) or "linear".
- simple** logical, with default TRUE, indicating whether or not simplified circle-arc equating should be used (see below).
- reps** the number of replications to use in bootstrapping. Passed to [bootstrap](#).
- xp, yp** optional parametric distributions, as frequency tables, replacing x and y when bootstrapping.
- xn, yn** sample sizes to be drawn from x and y, or xp and yp, at each bootstrap replication. These default to the observed sample sizes.
- crit** a vector of equated scores serving as the criterion equating function when calculating bootstrap bias and RMSE; both are returned when **crit** is specified.

General linear equating is a new approach to estimating a linear linking or equating function. The slope and intercept of the line are estimated based on multiple sources of information, including the means and standard deviations of X and Y, and other values supplied through **cx** and **cy**, representing the centrality of X and Y, and **sx** and **sy**, representing the scaling or variability of X and Y. The weights **wax** and **way** specify the proportional weighting given to the standard deviations of X and Y, and indirectly the weighting given to **sx** and **sy**, in determining the slope. **wbx** and **wby** specify the proportional weighting given to the means of X and Y, and indirectly the weighting given to **cx** and **cy**, in determining the intercept. Synthetic means and standard deviations will be used when appropriate. Chained general linear equating is not currently supported.

For equipercentile equating under the random groups design, three smoothing options are available: **smoothmethod = "average"** and **smoothmethod = "bump"** require the additional argument **jmin**, and loglinear smoothing (**smoothmethod = "loglinear"**) requires either a score function or maximum polynomial terms. For frequency estimation and chained methods, only smoothing methods "bump" and "loglinear" are supported. See the [presmoothing](#) function for details and examples.

In equipercentile equating, the high point for y, i.e., **highp[2]**, is used to obtain form Y equivalents of form X scores with percentile ranks of 100. Typically this is set to be the number of score points in the form Y scale, which assumes that scores are integers ranging from 1 (or 0) to the total number of items, and that each item is scored correct/incorrect. Scores on other scales (such as scales which include negative values, or which exclude zero) may also be used. In such cases **highp[2]** can be set to the highest possible score on form Y, or alternatively the highest observed score on Y.

**lowp** and **highp** are used to define the slope and intercept of the identity linking function. When the score scales for X and Y are equivalent, the identity function is simply the unequated X scale; however, when forms differ in their scales, e.g., because of changes in content or length, the identity linking function will map X onto Y based on the low and high coordinates.

The simplified approach to circle-arc equating, as demonstrated by Livingston and Kim (2009), involves combining a circle-arc with the identity function. When the low and high scores differ for the X and Y scales, this becomes the identity linking function. The linear component can be omitted, and symmetric circle-arc equating used, with **simple = FALSE**. The result is an equating function based only on the circle-arc that passes through the points **lowp**, **highp**, and the estimated midpoint.

Analytical standard errors are currently returned for linear equating under equivalent groups and chained, Tucker, and Levine equating with nonequivalent groups. Chained, Tucker, and Levine

standard errors are provided with and without assumptions of normality, as shown in Zu (2012). With `boot = TRUE`, bootstrap standard errors are estimated using a default of `reps = 100` replications, sampling the maximum amount from each score distribution (controlled by the arguments `xn` and `yn`). See [bootstrap](#) for details and examples, including how to obtain bootstrap bias and RMSE.

### Value

When `y` contains output from an equating, a vector of equated scores is returned. Otherwise, an object of class “`equate`” is returned, listing the following components, some of which are dependent on the equating type, method, and smoothing:

<code>name</code>	name for the equating
<code>type</code>	equating type
<code>method</code>	equating method
<code>design</code>	equating design, as specified in <code>x</code>
<code>x, y</code>	original frequency tables for <code>X</code> and <code>Y</code>
<code>concordance</code>	conversion table containing scores on <code>X</code> with their form <code>Y</code> equivalents, and standard errors, when available.
<code>points</code>	low and high points defining the identity line, and midpoints for general linear and circle-arc equating
<code>weight</code>	weights used in general linear equating
<code>internal, lts, jmin, degree, xdegree, scorefun</code>	additional arguments, as supplied in ...
<code>coefficients</code>	conversion coefficients intercept and slope; for circle-arc equating, circle center points and radius are also included; for general linear equating, slope and intercept components are included
<code>ws</code>	weight applied to <code>X</code> in synthetic estimation
<code>synthstats</code>	means and standard deviations for the synthetic distributions
<code>xsynthetic, ysynthetic</code>	frequency tables for the synthetic distributions
<code>smoothmethod</code>	smoothing method
<code>xsmooth, ysmooth</code>	smoothed frequency tables for <code>X</code> and <code>Y</code>
<code>bootstraps</code>	list containing bootstrap standard errors, and, optionally, other bootstrap output

The summary method returns a list with the name, type, method, design, and synthetic weight, along with frequency tables for the total, anchor, and equated score distributions and descriptive statistics for each.

### Methods (by class)

- `equate(list)`: Equating a list of frequency tables.
- `equate(freqlist)`: Equating frequency distributions in `x` and `y`.
- `equate(default)`: Default equating method for a vector of raw scores `x` and equating output in `y`.

**Author(s)**

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**References**

Albano, A. D. (2016). equate: An R package for observed-score linking and equating. *Journal of Statistical Software*, 74(8), 1–36.

Kolen, M. J., and Brennan, R. L. (2004). *Test Equating, Scaling, and Linking*. (2nd ed.), New York: Springer.

Livingston, S. A., and Kim, S. (2009). The circle-arc method for equating in small samples, *Journal of Educational Measurement*, 46, 330–343.

Zu, J., and Yuan, K. H. (2012). Standard error of linear observed-score equating for the NEAT design with nonnormally distributed data. *Journal of Educational Measurement*, 49, 190–213.

**See Also**

[freqbump](#), [freqavg](#), [loglinear](#), [bootstrap](#)

**Examples**

```
# See vignette("equatevignette") and Albano (2016) for a
# description of methods and additional examples

# Random groups equating for (1) identity, (2) mean,
# (3) linear, (4) equipercentile with loglinear
# smoothing, and (5) a composite of mean and identity
rx <- as.freqtab(ACTmath[, 1:2])
ry <- as.freqtab(ACTmath[, c(1, 3)])
set.seed(2007)

req1 <- equate(rx, ry, type = "i", boot = TRUE, reps = 5)
req2 <- equate(rx, ry, type = "m", boot = TRUE, reps = 5)
req3 <- equate(rx, ry, type = "l", boot = TRUE, reps = 5)
req4 <- equate(rx, ry, type = "e", boot = TRUE, reps = 5,
  smooth = "loglin", degree = 3)
req5 <- composite(list(req1, req2), wc = .5, symmetric = TRUE)

# Compare equating functions
plot(req1, req2, req3, req4, req5[[1]], addident = FALSE)

# Compare bootstrap standard errors
# Bootstrapping isn't supported for composite equating
plot(req1, req2, req3, req4, addident = FALSE, out = "se",
  legendplace = "topleft")

# Nonequivalent groups design for (1) Tucker linear,
# (2) frequency estimation, and (3) Braun/Holland linear
nx <- freqtab(KBneat$x, scales = list(0:36, 0:12))
ny <- freqtab(KBneat$y, scales = list(0:36, 0:12))
```

```

neq1 <- equate(nx, ny, type = "linear", method = "tuck", ws = 1)
neq2 <- equate(nx, ny, type = "equip", method = "freq", ws = 1)
neq3 <- equate(nx, ny, type = "linear", method = "braun", ws = 1)

# Compare equated scores
round(cbind(xscale = 0:36, tucker = neq1$conc$yx,
  fe = neq2$conc$yx, braun = neq3$conc$yx), 2)

# Multiple linkings using PISA reading booklet 6
# clusters 3a, 5, 6, and 7
r3 <- freqtab(PISA$totals$b6$r3a, scales = 0:15)
r5 <- freqtab(PISA$totals$b6$r5, scales = 0:15)
r6 <- freqtab(PISA$totals$b6$r6, scales = 0:15)
r7 <- freqtab(PISA$totals$b6$r7, scales = 0:14)
eqargs <- list(r3r5 = list(type = "linear", x = "r3", y = "r5",
  name = "Linear Linking PISA r3 to r5"),
  r5r6 = list(type = "linear", x = "r5", y = "r6",
  name = "Linear Linking PISA r5 to r6"),
  r6r7 = list(type = "linear", x = "r6", y = "r7",
  name = "Linear Linking PISA r6 to r7"))
req <- equate(list(r3 = r3, r5 = r5, r6 = r6, r7 = r7), eqargs)

# Put PISA r3 on the scale of r7 using the linking chain
# Compare to a direct linking of r3 to r7
equate(equate(req$r3r5$conc$yx, req$r5r6), req$r6r7)
equate(r3, r7, "linear")$conc$yx

# Linking PISA cluster r3a to r5 with multiple anchors
m367 <- freqtab(PISA$totals$b6[1:198, c("r3a", "r6", "r7")],
  scales = list(0:15, 0:16, 0:14))
m567 <- freqtab(PISA$totals$b6[199:396, c("r5", "r6", "r7")],
  scales = list(0:15, 0:16, 0:14))
meq1 <- equate(m367, m567, type = "mean", method = "nom")
meq2 <- equate(m367, m567, type = "mean", method = "tuck")
meq3 <- equate(m367, m567, type = "lin", method = "tuck")
meq4 <- equate(m367, m567, type = "equip", method = "freq",
  smooth = "log", show = FALSE)
meq <- equate(m367, m567, type = "mean", method = "nom")
plot(meq1, meq2, meq3, meq4, meq, req[[1]])

```

**Description**

Functions for creating and manipulating frequency tables of class "freqtab".

**Usage**

```

freqtab(x, ...)

## Default S3 method:
freqtab(x, scales, items, design, na.rm = TRUE, ...)

as.freqtab(x, scales, design, drop = FALSE, ...)

## S3 method for class 'table'
freqtab(x, design, ...)

## S3 method for class 'freqtab'
as.data.frame(x, row.names = NULL, optional = FALSE, drop = FALSE, ...)

## S3 method for class 'freqtab'
head(x, ...)

## S3 method for class 'freqtab'
tail(x, ...)

scales(x, margin = 1)

margin(x, margin = 1)

margins(x)

## S3 method for class 'freqtab'
droplevels(x, ...)

```

**Arguments**

<code>x</code>	either an object (vector or <code>data.frame</code> ) containing total scores or item responses with which total scores will be calculated, or an object inheriting from class “ <code>freqtab</code> ”. In the <code>freqtab</code> function, the first column in <code>x</code> must be the total test; any remaining columns may contain anchor scores. See below for details.
<code>...</code>	further arguments passed to or from other functions.
<code>scales</code>	list of vectors containing the score scales for each score scale in <code>x</code> .
<code>items</code>	list of vectors of column indices (numbers or column names) with which total scores will be computed for <code>x</code> when it contains item responses.
<code>design</code>	the equating design used in data collection. For univariate <code>x</code> , <code>design = "eg"</code> is assumed for equivalent groups. For multivariate <code>x</code> , <code>design = "ng"</code> is assumed for nonequivalent groups. Single-groups and counterbalanced designs must be specified with <code>design = "sg"</code> and <code>design = "cb"</code> .
<code>na.rm</code>	logical with default <code>TRUE</code> specifying whether or not missing item responses should be ignored, i.e., treated as 0, when calculating total scores.
<code>drop</code>	logical, with default <code>FALSE</code> , indicating whether or not unused factor levels, or score values with zero counts, should be dropped. See below for details.

`row.names`, optional  
arguments passed to `as.data.frame`, currently ignored.

`margin`  
integer vector specifying the margin(s) over which frequencies should be summed.

## Details

`freqtab` creates a frequency table from a vector or `data.frame` of scores. When the `items` argument is included, scores are assumed to be item responses, which are summed to create total scores. The scores are tabulated and stored as an array, with dimensions for each variable. Note that in previous versions of the “`freqtab`” class the frequency table was stored as a `data.frame`. This is no longer the case. Instead, the table is stored as an array and converted to a `data.frame` when printed or manipulated with the `head` and `tail` methods.

`as.data.frame` converts an object of class “`freqtab`” to “`data.frame`”. `droplevels` returns `x` with any unused factor levels, or levels with zero counts, removed.

When `x` is an object of class “`table`”, `freqtab` simply modifies the attributes and converts to class “`freqtab`”. In this case, `x` must already be structured similar to a “`freqtab`” object, with the first dimension containing counts for total scores, and remaining dimensions containing counts for one or more anchor tests.

`as.freqtab` converts a ‘flat’ contingency table (see [ftable](#)) to class “`freqtab`” with the appropriate attributes. A flat contingency table is the `data.frame` version of a “`freqtab`” object, where the first column contains the total score scale, the last column contains counts, and the columns in between contain different anchor test score combinations. `is.freqtab` tests for class “`freqtab`”.

`scales` extracts the measurement scales for the variables specified in `margin`, with `margin = 1` referring to the total score scale, and subsequent margins referring to anchor tests. `margin` is a wrapper for [margin.table](#), which itself is a simple wrapper for summing over marginal counts, i.e., `apply(x, margin, sum)`. And `margins` returns the number of dimensions, i.e., score variables, in a frequency table.

`design` is used to set the `dimnames` of the frequency table, with `total1` and `total2` used with single and counterbalanced groups, and `total` and `anchor(s)` used otherwise. `design` also sets the `design` attribute, which is used in [equate](#).

The main difference between the “`freqtab`” class and other tabulation classes, like “`table`” and “`ftable`”, is that the `dimnames`, i.e., the score scales, are required to be numeric. This facilitates plotting with [plot.freqtab](#), equating with the [equate](#) function, and descriptive statistics with the [summary.freqtab](#) and other methods.

## Value

A table array with dimensions equal to the number of score scales. In most cases, this will be a univariate or bivariate distribution, but multivariate distributions are supported. `scales` and `margins` return numeric vectors.

## Methods (by class)

- `freqtab(default)`: Default method for a `data.frame` of item responses, a `data.frame` of total and anchor scores, or a vector of total scores.
- `freqtab(table)`: Method for tables.



**Author(s)**

Anthony Albano <tony.d.albano@gmail.com>

**See Also**

[table](#), [ftable](#), [summary.freqtab](#), [plot.freqtab](#)

**Examples**

```
# Univariate distribution with score scale
set.seed(2005)
x <- round(rnorm(1000, 100, 10))
head(freqtab(x, scales = 70:130))

# Existing frequency table converted to class "freqtab"
# The first score of zero, with zero counts, is dropped
head(as.freqtab(ACTmath[, 1:2], drop = TRUE))

# Bivariate distribution
# Reduce y to the anchor test margin (2)
ny <- freqtab(x = KBneat$y, scales = list(0:36, 0:12))
margin(ny, margin = 2)

# Summing scored item responses with the PISA data
attach(PISA)
r6items <- paste(items$itemid[items$clusterid == "r6"])
r7items <- paste(items$itemid[items$clusterid == "r7"])
pisa67 <- freqtab(students[students$book == 6, ],
  items = list(r6items, r7items),
  scales = list(0:16, 0:14))
detach(PISA)

# Scales for both margins
# Zero total score is unobserved
scales(pisa67, 1:2)
scales(droplevels(pisa67), 1:2)
```

---

KBneat

*Test Scores under a NEAT design*


---

**Description**

This dataset contains scores for two forms of a 36-item test administered under a nonequivalent groups with anchor test design. The anchor test is internal and consists of twelve items taken by both groups of examinees. The data is distributed as part of the equating software CIPE (see link below), and additional examples using this dataset are presented in *Test Equating, Scaling, and Linking* (Kolen and Brennan, 2004).

**Usage**

KBneat

**Format**

A list of length two, where each element ( $\$x$  and  $\$y$ ) contains two columns of data, one for the total score and another for the anchor test score for each examinee.

**Source**

Kolen, M. J., & Brennan, R. L. (2004). *Test Equating, Scaling, and Linking*. (2nd ed.), New York: Springer.

The dataset can be downloaded as part of the CIPE software, available at the following link: <https://education.uiowa.edu/casma>

---

PISA

*Programme for International Student Assessment 2009 USA Data*

---

**Description**

This dataset contains scored cognitive item response data from the 2009 administration of the Programme for International Student Assessment (PISA), an international study of education systems. The data, and license under which they are released, are available online at <https://www.oecd.org/pisa/>.

**Usage**

PISA

**Format**

PISA is a list containing four elements. The first, `PISA$students`, is a `data.frame` containing 233 variables across 5233 individuals, with one row per individual. All but one variable come from the USA PISA data file "INT\_COG09\_S\_DEC11.txt". The remaining variable, language spoken at home, has been merged in from the student questionnaire file "INT\_STQ09\_DEC11.txt". Variable names match those found in the original files:

**list("stidstd")** Unique student ID (one for each of the 5233 cases);

**list("schoolid")** School ID (there are 165 different schools);

**list("bookid")** ID for the test booklet given to a particular student, of which there were 13;

**list("langn")** Student-reported language spoken at home, with 4466 students reporting English (indicated by code 313), 484 students reporting Spanish (with code 156) and 185 students reporting "another language" (code 859);

**list("m033q01")** Scored item-response data across the 189 items included in the general cognitive assessment, described below; and

- to** Scored item-response data across the 189 items included in the general cognitive assessment, described below; and
- list("s527q04t")** Scored item-response data across the 189 items included in the general cognitive assessment, described below; and
- list("pv1math")** PISA scale scores, referred to in the PISA technical documentation as "plausible values".
- to** PISA scale scores, referred to in the PISA technical documentation as "plausible values".
- list("pv5read5")** PISA scale scores, referred to in the PISA technical documentation as "plausible values".

Next, `PISA$booklets` is a `data.frame` containing 4 columns and 756 rows and describes the 13 general cognitive assessment booklets. Variables include:

- list("bookid")** The test booklet ID, as in `PISA$students`;
- list("clusterid")** ID for the cluster or item subset in which an item was placed; items were fully nested within clusters; however, each item cluster appeared in four different test booklets;
- list("itemid")** Item ID, matching the columns of `PISA$students`; each item appears in `PISA$booklets` four times, once for each booklet; and
- list("order")** The order in which the cluster was presented within a given booklet.

`PISA$items` is a `data.frame` containing 4 columns and 189 rows, with one row per item. Variables include:

- list("itemid")** Item ID, as in `PISA$booklets`
- list("clusterid")** Cluster ID, as in `PISA$booklets`
- list("max")** Maximum possible score value, either 1 or 2 points, with dichotomous scoring (max of 1) used for the majority of items; and
- list("subject")** The subject of an item, equivalent to the first character in `itemid` and `clusterid`.
- list("format")** Item format, abbreviated as `mc` for multiple choice, `cmc` for complex multiple choice, `ocr` for open constructed response, and `ccr` for closed constructed response.
- list("noptions")** Number of options, zero except for some multiple choice items.

Finally, `PISA$totals` is a list of 13 `data.frames`, one per booklet, where the columns correspond to total scores for all students on each cluster for the corresponding booklet. These total scores were calculated using `PISA$students` and `PISA$booklets`. Elements within the `PISA$totals` list are named by booklet, and the columns in the `data.frame` are named by cluster. For example, `PISA$totals$b1$m1` contains the total scores on cluster M1 for students taking booklet 1.

## Source

OECD (2012). PISA 2009 Technical Report, PISA, OECD Publishing. <http://dx.doi.org/10.1787/9789264167872-en>

Addition information can be found at the PISA website: <https://www.oecd.org/pisa/>

---

plot.bootstrap      *Plotting Bootstrap Equating Results*

---

### Description

This function plots bootstrap equating results for objects of class “bootstrap”.

### Usage

```
## S3 method for class 'bootstrap'
plot(
  x,
  add = FALSE,
  out = "mean",
  xpoints,
  ypoints,
  addident = TRUE,
  identy,
  identcol = 1,
  rescale = c(0, 1),
  xlab = "Total Score",
  ylab,
  col = rainbow(length(x$args)),
  pch,
  lty = 1,
  subset,
  morepars = NULL,
  addlegend = TRUE,
  legendtext,
  legendplace = "bottomright",
  ...
)
```

### Arguments

x	output from the <code>bootstrap</code> function.
add	logical, with default FALSE, specifying whether to create a new plot or add to the current one.
out	character vector specifying the output to be plotted, either the mean equated scores ("mean"), standard errors ("se"), bias ("bias"), or RMSE ("rmse").
xpoints, ypoints	optional vectors of the same length containing raw scores on forms X and Y, assuming a single group or equivalent groups design.
addident	logical, with default TRUE, for plotting the identity function. The result depends on out.

identity	vector of y coordinates for plotting the identity line. Defaults to the identity function when out = "eqs", otherwise, a horizontal line with intercept 0.
identcol	color used for plotting the identity line.
rescale	intercept and slope, with default 0 and 1, used to rescale all lines before plotting.
xlab, ylab, col, pch, lty	graphical parameters passed to par, with the lengths of col and lty recycled as necessary.
subset	vector for subsetting the output when multiple equating functions are included in x.
morepars	list of additional graphical parameters, excluding xlab, ylab, col, pch, lty.
addlegend	logical, with default TRUE, indicating whether or not a legend should be added.
legendtext	character vector of text to be passed to the legend argument of the legend function, defaulting to a combination of the equating types and methods specified in each equating object.
legendplace	placement of the legend.
...	further arguments passed to or from other methods, excluding graphical parameters.

### Details

Lines are plotted for the chosen output type, whether mean equated scores across replications (out = "mean"), standard errors (out = "se"), bias (out = "bias") or RMSE (out = "rmse"). The result is similar to that of [plot.equate](#).

### Author(s)

Anthony Albano <tony.d.albano@gmail.com>

### See Also

[bootstrap](#), [plot.equate](#)

### Examples

```
set.seed(122713)
neat.x <- freqtab(KBneat$x, scales = list(0:36, 0:12))
neat.y <- freqtab(KBneat$y, scales = list(0:36, 0:12))
eqargs <- list(m.t = list(type = "mean", method = "t"),
  l.t = list(type = "lin", method = "t"),
  c.t = list(type = "circ", method = "t"))
bootout <- bootstrap(x = neat.x, y = neat.y, args = eqargs,
  reps = 20)
plot(bootout, out = "se", legendplace = "top")
```

---

plot.equate

*Plotting Equating Results*


---

### Description

Functions for plotting equating functions from one or more objects of class “equate” or “equate.list”.

### Usage

```
## S3 method for class 'equate'
plot(
  ...,
  elist = NULL,
  add = FALSE,
  out = "eqs",
  xpoints,
  ypoints,
  addident = TRUE,
  identy,
  identcol = 1,
  rescale = c(0, 1),
  xlab = "Total Score",
  ylab,
  col = rainbow(length(x)),
  pch,
  lty = 1,
  lwd = 1,
  subset,
  morepars = NULL,
  addlegend = TRUE,
  legendtext,
  legendplace = "bottomright"
)

## S3 method for class 'equate.list'
plot(x, ...)
```

### Arguments

...	one or more equating objects, each containing results for equating the same two test forms.
elist	list of equatings to be plotted.
add	logical, with default FALSE, specifying whether to create a new plot or add to the current one.
out	character vector specifying the output to be plotted, either equating functions ("eqs"), standard errors ("se"), bias ("bias"), or RMSE ("rmse").

xpoints, ypoints	optional vectors of the same length containing raw scores on forms X and Y, assuming a single group or equivalent groups design.
addident	logical, with default TRUE, for plotting the identity function. The result depends on out.
identy	vector of y coordinates for plotting the identity line. Defaults to the X scale when out = "eqs", otherwise, a horizontal line with intercept 0.
identcol	color used for plotting the identity line.
rescale	intercept and slope, with default 0 and 1, used to rescale all lines before plotting.
xlab, ylab, col, pch, lty, lwd	graphical parameters passed to par, with col, pch, lty, and lwd recycled as necessary.
subset	vector for subsetting the output when multiple equating functions are included in x.
morepars	list of additional graphical parameters, excluding xlab, ylab, col, pch, lty, and lwd.
addlegend	logical, with default TRUE, indicating whether or not a legend should be added.
legendtext	character vector of text to be passed to the legend argument of the legend function, defaulting to a combination of the equating types and methods specified in each equating object.
legendplace	placement of the legend.
x	" <a href="#">equate.list</a> " object, containing output from multiple equatings.

## Details

Equating functions (out = "eqs") are plotted as lines based on the concordance table for each equating object that is supplied. Standard errors (out = "se") default to bootstrap standard errors, if available, otherwise, analytical standard errors are plotted. Bias (out = "bias") and RMSE (out = "rmse") are also taken from bootstrapping output.

## Examples

```
# See ?equate for additional examples

rx <- as.freqtab(ACTmath[, 1:2])
ry <- as.freqtab(ACTmath[, c(1, 3)])
set.seed(2007)

req1 <- equate(rx, ry, type = "i", boot = TRUE, reps = 5)
req2 <- equate(rx, ry, type = "m", boot = TRUE, reps = 5)
req3 <- equate(rx, ry, type = "l", boot = TRUE, reps = 5)
req4 <- equate(rx, ry, type = "e", boot = TRUE, reps = 5,
  smooth = "loglin", degree = 3)
req5 <- composite(list(req1, req2), wc = .5, symmetric = TRUE)

plot(req1, req2, req3, req4, req5[[1]], addident = FALSE)
plot(req5)
```

plot.freqtab

*Plotting Frequency Distributions***Description**

This function plots univariate and bivariate frequency tables of class “[freqtab](#)”.

**Usage**

```
## S3 method for class 'freqtab'
plot(
  x,
  y = NULL,
  xcol = 1,
  ycol,
  pch = 16,
  ylty = 1,
  xlab = names(dimnames(x))[1],
  addlegend = !missing(y),
  legendtext,
  ...
)

## S3 method for class 'freqtab'
points(x, xcol = 1, pch = 16, ds = 50, dm = 100, ...)
```

**Arguments**

<code>x</code>	univariate or bivariate score distribution of class “ <a href="#">freqtab</a> ”.
<code>y</code>	either an object of class “ <a href="#">freqtab</a> ”, where frequencies will be extracted, or a vector or matrix of frequencies, to be added to the plot of <code>x</code> . See below for details.
<code>xcol, ycol</code>	colors used in plotting <code>x</code> and <code>y</code> .
<code>pch</code>	plotting symbol used to plot bivariate points.
<code>ylty</code>	line type used to plot frequencies in <code>y</code> .
<code>xlab</code>	label for the <code>x</code> axis.
<code>addlegend</code>	logical indicating whether or not a legend should be added.
<code>legendtext</code>	character vector of text to be passed to the legend argument of the legend function, defaulting to column names used in <code>y</code> .
<code>...</code>	further arguments passed to or from other methods, such as graphical parameters besides <code>col</code> , <code>type</code> , and <code>pch</code> .
<code>ds, dm</code>	integers for the scaling and center of the RGB density values, with defaults of 50 and 100. These are used to convert the observed counts in <code>x</code> to the [0, 255] range of RGB values.



**Details**

For the points method, a scatterplot for x is added to the current opened plot.

For the plot method, when x is univariate, i.e. having 2 columns, a frequency plot is created for x. When x is bivariate, e.g., coming from a single group equating design or one form of a nonequivalent groups design, a scatterplot is produced with frequency plots for the marginal distributions.

y is used to superimpose lines, e.g., smoothed frequencies, over the (marginal) frequencies of x.

Colors must be specified using xcol and ycol. When ycol is missing, a vector of colors is created using rainbow(ncol(y)).

**Value**

The univariate option produces a single line plot of type = "h". Frequencies from y are then superimposed. The bivariate option produces a scatterplot with a marginal frequency plot for each distribution.

**Author(s)**

Anthony Albano <tony.d.albano@gmail.com>

**See Also**

[plot.table](#), [plot.equate](#), [lines](#), [points](#)

**Examples**

```
x <- freqtab(KBneat$x, scales = list(0:36, 0:12))
plot(x)
```

```
xs <- loglinear(x, degrees = c(4, 1),
  stepup = TRUE, showWarnings = FALSE)
plot(x, xs, lwd = 2)
```

**Description**

These functions are used to smooth frequency distributions.

**Usage**

```

presmoothing(x, ...)

## Default S3 method:
presmoothing(
  x,
  smoothmethod = c("none", "average", "bump", "loglinear"),
  jmin,
  asfreqtab = TRUE,
  ...
)

## S3 method for class 'formula'
presmoothing(x, data, ...)

loglinear(
  x,
  scorefun,
  degrees = list(4, 2, 2),
  grid,
  rmimpossible,
  asfreqtab = TRUE,
  models,
  stepup = !missing(models),
  compare = FALSE,
  choose = FALSE,
  choosemethod = c("chi", "aic", "bic"),
  chip,
  verbose = FALSE,
  ...
)

freqbump(x, jmin = 1e-06, asfreqtab = FALSE, ...)

freqavg(x, jmin = 1, asfreqtab = FALSE, ...)

```

**Arguments**

x	either an object of class “freqtab” specifying a univariate or multivariate score distribution, or a “formula” object.
...	further arguments passed to other methods. For presmoothing, these are passed to loglinear and include those listed above.
smoothmethod	character string indicating the smoothing method to be used by presmoothing. “none” returns unsmoothed frequencies, “bump” adds a small frequency to each score value, “average” imputes small frequencies with average values, and “loglinear” fits loglinear models. See below for details.
jmin	for smoothmethod = “average”, the minimum frequency, as an integer, below

	which frequencies will be replaced (default is 1). for smoothmethod = "bump", the value to be added to each score point (as a probability, with default 1e-6).
asfreqtab	logical, with default TRUE, indicating whether or not a frequency table should be returned. For smoothmethod = "average" and smoothmethod = "bump", the alternative is a vector of frequencies. For loglinear, there are other options.
data	an object of class "freqtab".
scorefun	matrix of score functions used in loglinear presmoothing, where each column includes a transformation of the score scale or interactions between score scales. If missing, degrees and xdegree will be used to construct polynomial score functions.
degrees	list of integer vectors, each one indicating the maximum polynomial score transformations to be computed for each variable at a given order of interactions. Defaults (degrees = list(4, 2, 2)) are provided for up to trivariate interactions. degrees are ignored if scorefun or grid are provided. See below for details.
grid	matrix with one column per margin in x and one row per term in the model. See below for details.
rmimpossible	integer vector indicating columns in x to be used in removing impossible scores before smoothing, assuming internal anchor variables. Impossible scores are kept by default. See below.
models	integer vector indicating which model terms should be grouped together when fitting multiple nested models. E.g., models = c(1, 1, 2, 3) will compare three models, with the first two terms in model one, the third term added in model two, and the fourth in model three.
stepup	logical, with default FALSE, indicating whether or not multiple nested models should be automatically fit. If TRUE and models is missing, an attempt will be made to create it using grid and/or degrees. Otherwise, in the absence of models, each column in scorefun will define a new sequential model.
compare	logical, with default FALSE, indicating whether or not fit for nested models should be compared. If TRUE, stepup is also set to TRUE and only results from the model fit comparison are returned, that is, verbose is ignored.
choose	logical, with default FALSE, indicating whether or not the best-fitting model should be returned after comparing fit of nested models. Useful for automating model selection in simulations.
choosemethod	string, indicating the method for selecting a best-fitting model when choose = TRUE. "chi" selects the most complex model with chi-square p-value below the criterion in chip. Remaining methods choose the model with lowest value.
chip	proportion specifying the type-I error rate for model selection based on choosemethod = "chi".
verbose	logical, with default FALSE, indicating whether or not full glm output should be returned.

## Details

Loglinear smoothing is a flexible procedure for reducing irregularities in a frequency distribution prior to equating, where the degree of each polynomial term determines the specific moment of

the observed distribution that is preserved in the fitted distribution (see below for examples). The `loglinear` function is a wrapper for `glm`, and is used to simplify the creation of polynomial score functions and the fitting and comparing of multiple loglinear models.

`scorefun`, if supplied, must contain at least one score function of the scale score values. Specifying a list to `degrees` is an alternative to supplying `scorefun`. Each list element in `degrees` should be a vector equal in length to the number of variables contained in `x`; there should also be one such vector for each possible level of interaction between the variables in `x`.

For example, the default `degrees = list(4, 2, 2)` is recycled to produce `list(c(4, 4, 4), c(2, 2, 2), c(2, 2, 2))`, resulting in polynomials to the fourth power for each univariate distribution, to the second power for each two-way interaction, and to the second power for the three-way interaction.

Terms can also be specified with `grid`, which is a matrix with each row containing integers specifying the powers for each variable at each interaction term, including main effects. For example, the main effect to the first power for the total score in a bivariate distribution would be `c(1, 0)`; the interaction to the second power would be `c(2, 2)`.

`stepup` is used to run nested models based on subsets of the columns in `scorefun`. Output will correspond to models based on columns 1 and 2, 1 through 3, 1 through 4, to 1 through `ncol(scorefun)`. This list of polynomial terms is then used to create a `grid` using `expand.grid`. The `grid` can also be supplied directly, in which case `degrees` will be ignored.

`compare` returns output as an anova table, comparing model fit for all the models run with `stepup = TRUE`, or by specifying more than one model in `models`. When `choose = TRUE`, the arguments `choosemethod` and `chip` are used to automatically select the best-fitting model based on the anova table from running `compare`.

The remaining smoothing methods make adjustments to scores with low or zero frequencies. `smoothmethod = "bump"` adds the proportion `jmin` to each score point and then adjusts the probabilities to sum to 1. `smoothmethod = "average"` replaces frequencies falling below the minimum `jmin` with averages of adjacent values.

## Value

When `smoothmethod = "average"` or `smoothmethod = "bump"`, either a smoothed frequency vector or table is returned. Otherwise, `loglinear` returns the following:

- when `compare = TRUE`, an anova table for model fit
- when `asfreqtab = TRUE`, a smoothed frequency table
- when `choose = TRUE`, a smoothed frequency table with attribute "anova" containing the model fit table for all models compared
- when `verbose = TRUE`, full `glm` output, for all nested models when `stepup = TRUE`
- when `stepup = TRUE` and `verbose = FALSE`, a `data.frame` of fitted frequencies, with one column per model

## Methods (by class)

- `presmoothing(default)`: Default method for frequency tables.
- `presmoothing(formula)`: Method for "formula" objects.

**Author(s)**

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**References**

Holland, P. W., and Thayer, D. T. (1987). *Notes on the use of log-linear models for fitting discrete probability distributions* (PSR Technical Rep. No. 87-79; ETS RR-87-31). Princeton, NJ: ETS.

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Moses, T., and Holland, P. W. (2008). *Notes on a general framework for observed score equating* (ETS Research Rep. No. RR-08-59). Princeton, NJ: ETS.

Moses, T., and Holland, P. W. (2009). Selection strategies for univariate loglinear smoothing models and their effect on equating function accuracy. *Journal of Educational Measurement*, 46, 159–176. ETS.

Wang, T. (2009). Standard errors of equating for the percentile rank-based equipercntile equating with log-linear presmoothing. *Journal of Educational and Behavioral Statistics*, 34, 7–23.

**See Also**

[glm](#), [loglin](#)

**Examples**

```
set.seed(2010)
x <- round(rnorm(1000, 100, 15))
xscale <- 50:150
xtab <- freqtab(x, scales = xscale)

# Adjust frequencies
plot(xtab, y = cbind(average = freqavg(xtab),
  bump = freqbump(xtab)))

# Smooth x up to 8 degrees and choose best fitting model
# based on aic minimization
xlog1 <- loglinear(xtab, degrees = 8,
  choose = TRUE, choosemethod = "aic")
plot(xtab, as.data.frame(xlog1)[, 2],
  legendtext = "degree = 3")

# Add "teeth" and "gaps" to x
# Smooth with formula interface
teeth <- c(.5, rep(c(1, 1, 1, 1, .5), 20))
xttab <- as.freqtab(cbind(xscale, c(xtab) * teeth))
xlog2 <- presmoothing(~ poly(total, 3, raw = TRUE),
  xttab, showWarnings = FALSE)

# Smooth xt using score functions that preserve
# the teeth structure (also 3 moments)
teeth2 <- c(1, rep(c(0, 0, 0, 0, 1), 20))
xt.fun <- cbind(xscale, xscale^2, xscale^3)
```

```

xt.fun <- cbind(xt.fun, teeth2, xt.fun * teeth2)
xlog3 <- loglinear(xttab, xt.fun, showWarnings = FALSE)

# Plot to compare teeth versus no teeth
op <- par(no.readonly = TRUE)
par(mfrow = c(3, 1))
plot(xttab, main = "unsmoothed", ylim = c(0, 30))
plot(xlog2, main = "ignoring teeth", ylim = c(0, 30))
plot(xlog3, main = "preserving teeth", ylim = c(0, 30))
par(op)

# Bivariate example, preserving first 3 moments of total
# and anchor for x and y, and the covariance
# between anchor and total
# see equated scores in Wang (2009), Table 4
xvtab <- freqtab(KBneat$x, scales = list(0:36, 0:12))
yvtab <- freqtab(KBneat$y, scales = list(0:36, 0:12))
Y <- as.data.frame(yvtab)[, 1]
V <- as.data.frame(yvtab)[, 2]
scorefun <- cbind(Y, Y^2, Y^3, V, V^2, V^3, V*Y)
wang09 <- equate(xvtab, yvtab, type = "equip",
  method = "chained", smooth = "loglin",
  scorefun = scorefun)
wang09$concordance

# Removing impossible scores has essentially no impact
xvlog1 <- loglinear(xvtab, scorefun, asfreqtab = FALSE)
xvlog2 <- loglinear(xvtab, scorefun, rmimpossible = 1:2)
plot(xvtab, cbind(xvlog1,
  xvlog2 = as.data.frame(xvlog2)[, 3]))

```

---

px

*Percentile Ranks and Cumulative Frequencies*


---

## Description

These functions compute percentile ranks and cumulative frequency distributions for frequency tables.

## Usage

```

px(x, ...)

## Default S3 method:
px(x, y, ys, ...)

## S3 method for class 'freqtab'
px(x, margin = 1, y, ymargin = 1, ...)

```

```

fx(x, ...)

## Default S3 method:
fx(x, ...)

## S3 method for class 'freqtab'
fx(x, margin = 1, ...)

```

### Arguments

x	either a vector of counts, or an object of class “freqtab” from which counts will be taken.
...	further arguments passed to or from other methods.
y	an object of class “freqtab” when x is as well, otherwise, a vector or data.frame of counts. See below for details.
ys	vector specifying the y score scale, when it is not contained in the first column of y. If y can be converted to a data.frame, it is assumed to be univariate with the first column containing the score scale and the second containing the counts.
margin, ymargin	integers specifying the margins for which frequencies or percentile ranks will be returned. margin applies to x and ymargin to y.

### Details

These functions compute percentile ranks and cumulative frequencies for a univariate distribution, and percentile ranks from one univariate distribution (x) corresponding to score values in another (y).

### Value

A vector is returned containing either percentile ranks or cumulative frequencies with length equal to length(x).

### Author(s)

Anthony Albano <tony.d.albano@gmail.com>

### See Also

[freqtab](#)

### Examples

```

x <- as.freqtab(ACTmath[, 1:2], drop = TRUE)
y <- as.freqtab(ACTmath[, c(1, 3)], drop = TRUE)

# Percentile ranks for the x scale
round(px(x), 3)

```

```
# Percentile ranks in y for x each score
round(px(x, y = y), 3)

# Cumulative frequency distribution for x
round(fx(x), 3)
```

---

sample.freqtab	<i>Bootstrap Random Sampling from Frequency Tables</i>
----------------	--

---

### Description

An extension of [sample](#) to objects of class “freqtab” for bootstrap sampling.

### Usage

```
sample.freqtab(x, size = sum(x), replace = TRUE)
```

### Arguments

x	object of class “freqtab”, which is an array of counts across one or more numeric dimensions.
size	non-negative integer giving the sample size.
replace	logical with default TRUE indicating whether sampling should be with replacement.

### Value

A table array, as a “freqtab” object, sampled from the original x.

### Author(s)

Anthony Albano <tony.d.albano@gmail.com>

### See Also

[table](#), [ftable](#), [summary.freqtab](#), [plot.freqtab](#)

### Examples

```
# Sample with replacement from ACT math and compare results
set.seed(2021)
rx <- as.freqtab(ACTmath[, 1:2])
rxs <- sample.freqtab(rx)
summary(rx)
summary(rxs)
```



---

summary.freqtab	<i>Descriptive Statistics for Frequency Tables</i>
-----------------	--

---

**Description**

These functions return descriptive statistics for a frequency table of class “freqtab”.

**Usage**

```
## S3 method for class 'freqtab'
summary(object, margin = seq(margins(object)), ...)
```

```
## S3 method for class 'freqtab'
mean(x, margin = 1, ...)
```

```
sd.freqtab(x, margin = 1)
```

```
var.freqtab(x, margin = 1)
```

```
cov.freqtab(x, margin = seq(margins(x)))
```

```
cor.freqtab(x, margin = seq(margins(x)))
```

```
## S3 method for class 'freqtab'
min(x, margin = 1, ..., na.rm = FALSE)
```

```
## S3 method for class 'freqtab'
max(x, margin = 1, ..., na.rm = FALSE)
```

```
## S3 method for class 'freqtab'
range(x, margin = 1, ..., na.rm = FALSE)
```

```
skew.freqtab(x, margin = 1)
```

```
kurt.freqtab(x, margin = 1)
```

**Arguments**

object, x	object of class “freqtab”.
margin	integer vector specifying the margin(s) for which summary statistics will be returned. This defaults to 1 for univariate statistics, and seq(margins(x)), i.e., all the margins, for multivariate statistics (covariance and correlation).
...	further arguments passed to or from other methods.
na.rm	logical indicating whether missing values should be removed, currently ignored since frequency tables cannot contain missing values.

**Details**

mean, sd.freqtab, var.freqtab, skew.freqtab, and kurt.freqtab return the mean, standard deviation, variance, skewness, and kurtosis. min and max return the minimum and maximum observed scores, and range returns both. cov.freqtab and cor.freqtab return the covariance and correlation matrices for one or more variables. summary returns univariate statistics across one or more margins.

**Value**

summary returns a data frame of summary statistics, including the mean, standard deviation, skewness, kurtosis, minimum, maximum, and number of observations for each variable in margin. Otherwise, a vector of length length(margin) is returned with the corresponding statistic for each variable.

**Author(s)**

Anthony Albano <tony.d.albano@gmail.com>

**See Also**

[freqtab](#)

**Examples**

```
summary(as.freqtab(ACTmath[, 1:2]))

ny <- freqtab(KBneat$y, scales = list(0:36, 0:12))
summary(ny)
cov.freqtab(ny)
```

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